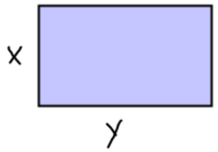
① Compare the two following problems and complete them as far as you can.

ⓐ Find the area of the rectangle when $x=24$

$A = 24y\ u^2$



ⓑ Find the area of the rectangle when $x=24$ and $\frac{y}{x} = \frac{3}{2}$.

$A = 864\ u^2$

② Find the equations for the tangent and normal lines to the curve $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$ at the point $(-1, 0)$.

$$12x + 3\left(x\frac{dy}{dx} + y\right) + 4y\frac{dy}{dx} + 17\frac{dy}{dx} = 0$$

$$12x + 3x\frac{dy}{dx} + 3y + 4y\frac{dy}{dx} + 17\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(3x + 4y + 17) = -12x - 3y$$

$$\frac{dy}{dx} = \frac{-12x - 3y}{3x + 4y + 17}$$

$$\left.\frac{dy}{dx}\right|_{(-1,0)} = \frac{-12(-1) - 3(0)}{3(-1) + 4(0) + 17}$$

$$= \frac{12}{14} = \frac{6}{7}$$

tangent $y = \frac{6}{7}(x+1)$

$y = \frac{6}{7}x + \frac{6}{7}$

Normal: $y = \frac{-7}{6}(x+1)$

Implicit Differentiation–

Differentiation takes place with respect to x. When you differentiate terms involving x alone, you can differentiate as usual. However, when you differentiate terms involving y, you must use the chain rule because you are assuming that y is defined implicitly as a differentiable function of x.

$$\frac{d}{dx} x^3 = 3x^2 \frac{dx}{dx}$$

variables agree

as normal

$$\frac{d}{dx} \overbrace{y^3}^{u^n} = \overbrace{3y^2}^{nu^{n-1}} \cdot \overbrace{\frac{dy}{dx}}^{u'}$$

variables disagree

use chain rule

A pebble is dropped into a calm pond, causing ripples to form in concentric circles. The length of the radius of the outer ripple is a function of the time since the pebble was dropped, t. If the outer ripple is increasing at a constant rate of 1 ft/sec, what is the rate of change of the area, A, when the radius is 4 feet?

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(4)\cdot(1) = 8\pi \text{ ft}^2/\text{sec}$$

3.7 #25

$y = 2\sin(\pi x - y)$ at $(1,0)$

$1\frac{dy}{dx}$

$$\frac{dy}{dx} = 2\cos(\pi x - y)\cdot\left(\pi - \frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = 2\pi\cos(\pi x - y) - 2\cos(\pi x - y)\frac{dy}{dx}$$

$$\frac{dy}{dx} + 2\cos(\pi x - y)\frac{dy}{dx} = 2\pi\cos(\pi x - y)$$

$$\frac{dy}{dx}\left(1 + 2\cos(\pi x - y)\right) = 2\pi\cos(\pi x - y)$$

$$\boxed{\frac{dy}{dx} = \frac{2\pi\cos(\pi x - y)}{1 + 2\cos(\pi x - y)}}$$

$$\left.\frac{dy}{dx}\right|_{(1,0)} = \frac{2\pi\cos(\pi(1) - 0)}{1 + 2\cos(\pi(1) - 0)}$$

$$= \frac{-2\pi}{\underset{-1}{1-2}} = 2\pi$$

tangent $= 2\pi(x-1)$

Normal $= -\frac{1}{2\pi}(x-1)$

3.7 #22

$$x^2 - \sqrt{3}xy + 2y^2 = 5 \quad \text{at } (\sqrt{3}, 2)$$

$$2x - \sqrt{3}\left(x\frac{dy}{dx} + y\right) + 4y\frac{dy}{dx} = 0$$

$$2x - \sqrt{3}x\frac{dy}{dx} - \sqrt{3}y + 4y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(-\sqrt{3}x + 4y\right) = \sqrt{3}y - 2x$$

$$\frac{dy}{dx} = \frac{\sqrt{3}y - 2x}{4y - \sqrt{3}x} \qquad \left.\frac{dy}{dx}\right|_{(\sqrt{3},2)} \frac{\sqrt{3}(2) - 2(\sqrt{3})}{4(2) - \sqrt{3}(\sqrt{3})} = \frac{0}{5} = 0$$

tangent $y = 0(x - \sqrt{3}) + 2$

$$\boxed{y = 2}$$

Normal $\boxed{x = \sqrt{3}}$

$\boxed{\text{HW}}$

- Sect. 3.7 # 29, 30, 34–42 (even), 43, 49, 51–53, 57
- Weekly Review 4 due Fri
- Test Next Mon.